Alternative derivation of anomaly polynomial

charge pairing in 4d:

$$q = (e, m) \quad , \quad q' = (e', m')$$

$$\rightarrow \langle q, q' \rangle_{4d} = \underbrace{em' - e'm}_{\text{anti-sym.}} \in \mathbb{Z}$$

in 6d:

For n self-dual tensor fields, there are self-dual strings (M-strings) with charges in n-dim lattice Λ

symmetric pairing: $\langle q, q' \rangle_{6d} = \langle q', q \rangle_{6d}$

$$\Big\downarrow T^2$$

4d charge : $q(\underbrace{mA + nB}_{\text{winding along } A\text{- and } B\text{-cycle of } T^2})$

$$\langle qA, q'B \rangle_{4d} = \langle q, q' \rangle_{6d} \underbrace{\langle A, B \rangle_{T^2}}_{\text{intersection number in } T^2}$$

Introduce $q = (q_i)_{i=1,\dots,n} \in \Lambda$

$$\rightarrow \langle q, q' \rangle_{6d} = \Omega^{ij} q_i q_j'$$

symmetric matrix

Introduce self-dual 3-form field strength H_i

$$dH_i = q_i \prod_{a=2,3,4,5} \delta(x_a)\, dx_a$$

for M-string of charge q at $x_{a=2,3,4,5} = 0$

$\rightarrow$ modification of Bianchi identity:

$$\boxed{dH_i = X_i} \qquad (*)$$

X_i: 4-form constructed of metric and gauge fields

gives contribution to anomaly polynomial:

<u>review</u>:

- descent formalism:

$$I_8 = dI_7, \quad \delta_\Lambda I_7 = dI_6'(\Lambda)$$

and $\delta_\Lambda S = \int I_6'(\Lambda)$

- $$\hat{I}_8 = \frac{10-\overset{\text{number of tensors}}{n}}{8}(\mathrm{tr}R^2)^2 + \frac{1}{6}\mathrm{tr}R^2 \sum_a X_a^{(2)} - \frac{2}{3}\sum_a X_a^{(4)} + 4\sum_{a<b} Y_{ab}$$

where $X_a^{(n)} = \mathrm{Tr}\, F_a^n - \sum_R n_{R_a} \mathrm{tr}_{R_a} F_a^n$

$$Y_{ab} = \sum_{R_a, R_b'} n_{R_a R_b'}\, \mathrm{tr}_{R_a} F_a^2\, \mathrm{tr}_{R_b'} F_b^2$$

notation: Tr : trace in adjoint rep.

tr_{R_a} : trace in rep. R_a

n_{R_a} : number of hypermultiplets

$n_{R_a R'_b}$: " " " in rep (R_a, R'_b) of $G_a \times G_b$

- anomaly cancellation:

I_8 should be representable as

$$I_8 = \frac{1}{2} \Omega_{ij} X^i X^j$$

with $X^i = \frac{1}{2} a^i \mathrm{tr} R^2 + 2 b^i_a \mathrm{tr} F_a^2$

(closed and gauge invariant)

cancellation due to "Green-Schwarz" mechanism:

$$A = \int_{\mathbb{R}^{5,1}} \Omega_{ij} B^i X^j$$

where $H^i = dB^i + \frac{1}{2} a^i \omega_{3L} + 2 b^i_a \omega^a_{3Y}$

gravitational Chern-Simons 3-form

Yang-Mills CS 3-form

$$\rightarrow \delta_\Lambda(A + S) = \int \Omega_{ij}(\delta_\Lambda B^i) X^j + \int I_6^1(\Lambda) = 0$$

Since $dH^i = X^i$, equation (*) gives:

$$I^{GS} = \frac{1}{2}\Omega^{ij} X_i X_j$$

6d Green-Schwarz and 5d Chern-Simons:

- S^1 reduction of $H_i \rightarrow n$ Abelian gauge fields A_i

$$\rightarrow F_{\mu\nu} = 2\pi R \cdot H_{\mu\nu 5}$$

$\rightarrow$ 5d Kinetic term: $\frac{1}{2R}\Omega^{ij} F_i \wedge * F_j$

and reduction of $dH_i = X_i$ gives

$$d\left(\frac{1}{2\pi R} * F_i\right) = X_i$$

$\rightarrow$ Chern-Simons term in 5d:

$$\frac{1}{2\pi} S^{CS} = \Omega^{ij} A_i X_j = A_i I^i$$

- consider a 5d fermion ψ with mass term $m\bar{\psi}\psi$ and charge q under a $U(1)$, coupling to non-Abelian background gauge field F_G in rep. ρ, coupling to metric

→ triangle diagrams give induced CS-term:

$$\frac{1}{2}(\text{sign } m)\, q A \left(\frac{1}{2}\mathrm{tr}_\rho F_G^2 + \frac{1}{24} d_\rho\, p_1(T)\right)$$

- Since reduction of 6d (2,0) theory of ADE type on T^2 gives 4d $\mathcal{N}=4$ with gauge group G of ADE
  → 6d charge lattice of M-strings is root lattice of G
  → Ω^{ij} is Cartan matrix η^{ij} of G

R-sym. of (2,0) theory is $SO(5)_R$
→ going to tensor branch gives $SO(4)_R$
  $SO(4)_R \simeq SU(2)_R \times SU(2)_L$
  by introducing vev $\phi \in \mathrm{Cartan}(\mathfrak{g})$
→ reduction on S^1 gives massive charged $\mathcal{N}=2$ vector multiplets of mass $|\phi \cdot \alpha|$ $\forall\, \alpha$ roots of $\mathfrak{g}$
→ pair of massive $\mathcal{N}=1$ VM and $\mathcal{N}=1$ HM
→ fermion masses $\psi \Gamma^{I} \phi^I \psi$

VM has mass $-\phi\cdot\alpha$, HM has mass $+\phi\cdot\alpha$

The induced CS terms are then:

$$\frac{1}{2}\sum_{\alpha>0}(\alpha\cdot A)\left[\left(c_2(L)+\frac{2}{24}p_1(T)\right)-\left(c_2(R)+\frac{2}{24}p_1(T)\right)\right]$$

$$=\rho\cdot A\left(c_2(L)-c_2(R)\right)$$

$\uparrow$

Weyl vector

Lifting back to 6d gives:

$$dH_i=\rho_i\left(c_2(L)-c_2(R)\right)$$

$\rightarrow$ GS contribution to anomaly of 6d theory:

$$\frac{1}{2}\langle\rho,\rho\rangle\left(c_2(L)-c_2(R)\right)^2=\frac{h_G^\vee d_G}{24}\left(c_2(L)-c_2(R)\right)^2$$

Using $p_2(N)=\left(c_2(L)-c_2(R)\right)^2$ gives then

$$I_G^{\mathcal{N}=(2,0)}=\frac{h_G^\vee d_G}{24}p_2(N)+r_G\, I^{\mathcal{N}=(2,0)\text{ tensor}}$$